

The Gauss Heritage

The astronomer, mathematician and geodesist Carl Friedrich Gauss died 150 years ago. He is famous for his contributions to mathematics, but more directly of interest in a GIS context is his work to determine the form and size of the geoid and his contribution to the foundation of the survey of Hanover (now part of Germany). Most impressive is the story of the teacher who, hoping for a quiet hour, asked his pupils to add the numbers from 1 to 100; in a very short space of time Gauss arrived at the answer 5,050 by computing $(1+100) + (2+99) + (3+98) \dots = 50 \cdot 101$.

This story perfectly illustrates the approach of one of the best scientists ever. Identify carefully what is important (here the set of numbers 1 to 100), ignore what is irrelevant (here the order of the numbers) and deduce a generally valid rule which states the sum of all integer numbers between 1 and n equals $\frac{n^2}{2} (n+1)$.

The same principles may be found in Gauss inventing (simultaneously with Legendre) the method of "least squares adjustment" and applying it to rediscover the planetoid Ceres in 1801. Gauss could use the then available few, and imperfect, observations to calculate the elements of the orbit, and the planet was found very close to where he had predicted it. Gauss used the same least squares adjustment methods to determine the distance between the astronomical observatory of Hamburg and Göttingen, where he was professor of astronomy, and for the adjustment of first-order trigonometric points. In this work he discovered surprisingly large deflections of the vertical that led him later to develop potential theory.

I was particularly impressed with his invention of the heliotrope: an instrument to reflect sunlight to a distant observer, helping him to point an instrument accurately in the direction to this point. Gauss found a geometric solution, namely two mirrors exactly at right angles, such that the light falling from the sun into the eye of the observer at A and the light going to the distant observer B form a straight line. An instrument based on this principle can be built and adjusted in the field to the precision necessary for aligning the instrument precisely to reflect sunlight to a point sixty miles away! Note that his aim was to describe a mathematical principle that could be used to build a useful instrument, not just to arrive at mathematical proof. The instrument was produced by instrument makers of the time and shown thus in a textbook of 1869.

Engineering science produces the most useful results if the scientist starts with profound practical experience and proceeds to thorough analysis whereby all irrelevant detail is discarded so as to identify the core of the question. A generalised solution may then become accessible, one that solves not only the problem at hand but also many future problems. The art demonstrated by Gauss was the most thorough analysis, leading him to higher levels of abstraction and thus to solutions that were more general than previous ones; so general, indeed, that many of his results survive up to the present day.